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THE PROBLEM OF THE DEVELOPMENT OF DROPLETS IN CLOUDS AND FOGS

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In 1939, Findeisen [1] proposed the theory that large cloud droplets are formed from the fusion of small droplets, produced by a difference in Stokes' fall velocities. He observed the fall of a drop through a cloud consisting of droplets of four different dimensions (radii of the droplets r = 5, 10, 15, 20 μ) and assumed the number of droplets of each size to be invariable. This conforms with the hypothesis of the infinitely slow formation of fine cloud droplets. When the water content in a cloud (the mass of water drops per unit volume) equals 1.25 gr/m^3 the radius of the droplet, according to Findeisen, increases from 30 to 100 μ during a fall of 150 meters and to 250 μ during a 550-meter fall.

In this article it is assumed that the droplets are distributed by size, in conformity with Schumann's formula [2], which gives an asymptotic form for Smoluchovskiy's distribution:

$$n(r) = \frac{25 q_w}{4\pi r_m^6} r^2 e^{-\frac{5}{3}\cdot\frac{r^3}{r_m^3}} \qquad (1)$$

Where q_w is the water content of the cloud or fog, r_m is the radius of the droplets forming the largest part of the water content.

Formula (1) is derived on the assumption that a change in the number of droplets of a given size occurs only as a result of fusion during collision (each collision leads to fusion!) but the number of collisions in a given volume per unit time does not depend upon the sizes of the droplets. The form of the distribution curve, despite the crudeness of the last hypothesis agrees rather closely with the

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experimental data of Houghton and Radford [3] who study fogs by the method of microphotography.

According to Schumann, the radius r_m in formula (1) increases upon the [illegible] of droplets as a result of differences in Stokes' fall velocities according to the formula:

$$r_m = \frac{r_{mo}}{1 - 3{,}23 \cdot 10^5 q \omega r_{mo} t}, \qquad (2)$$

Where r_{mo} is the value of r_m at the initial moment $t = 0$.

Let us use Schumann's results to calculate the increase in size of a droplet of any radius greater than r_m due to Stokes' fall velocities. Study of this problem is of especial importance in the problem of rain-origination because it is the enlarged droplets of a cloud which can grow to the size of raindrops, while the fine droplets (up to $r = r_m$) only furnish the material for their development. We shall not touch upon other effects in the formation of raindrops.

Let us calculate the droplet's rate of growth. For a droplet of radius r the number of collisions with droplets of radius r_1 during 1 second equals:

$$\nu = \sigma n(r_1) \Delta V, \qquad (3)$$

Where $\sigma = \pi (r + r_1)^2$ is the effective cross section of collision, $n(r_1)$ is the number of droplets of radius r_1 in one cubic centimeter, $\Delta V = \frac{2}{9}\frac{g}{\eta}(r^2 - r_1^2)$ is the difference in Stokes' fall velocities for the droplets, g is earth's acceleration, η is the viscosity coefficient of air.

The increase in the droplet's radius r during one collision with a droplet of small radius r_1 is given in the expression:

$$\Delta r = \frac{r_1^3}{3r^2} \qquad (4)$$

As collisions between enlarged droplets are very rare (by reason of the smallness of $n(r_1)$ and ΔV), we shall assume that a given enlarged droplet collides only with droplets of radii not greater than r_m.

The total change in the radius of a droplet per unit time because of collision with droplets of all sizes up to $r_1 = r_m$ will be equal to:

$$\frac{dr}{dt} = \frac{25 g q \omega}{54 \eta r_m^6 r^2} \int_0^{r_m} \left(r^4 r_1^5 + 2r^3 r_1^6 - 2r r_1^8 - r_1^9\right) e^{-\frac{5}{3} \cdot \frac{r_1^3}{r_m^3}} dr_1 \qquad (5)$$

After integration we shall obtain:

$$\frac{dr}{dt} = \frac{g q \omega}{18 \eta r^2} \Big\{ \left(1 - \frac{8}{3} e^{-5/3}\right) r^4 + 2\left(\frac{3}{5}\right)^{1/3} \gamma\left(\frac{7}{3}, \frac{5}{3}\right) r_m r^3 + \left(2 - \frac{73}{9} e^{-5/3}\right) r_m^3 r - \left[\frac{7}{5}\left(\frac{3}{5}\right)^{1/3} \gamma\left(\frac{7}{3}, \frac{5}{3}\right) - \frac{5}{3} e^{-5/3}\right] r_m^4 \Big\}, \qquad (6)$$

Where $\gamma(x, p)$ is the incomplete gamma-function tabulated by Pearson [4] and $\gamma(7/3, 5/3) = 0.473$.

As may be seen from (6), the rate of growth of an enlarged droplet increases with its radius. Making use of this fact, we shall calculate the growth of an enlarged droplet by successive steps in each of which we shall assume that r_m = constant.

The duration of each step must satisfy the condition that in the substitution for the right-hand term of equation (2) the second term is appreciably less than unity. Since, in actual clouds and fogs, r_m is of the order of several microns and in any case is not greater than several

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tens of microns, the criterion for the water content $q_w = 10^{-3}$ gr/cm^3 takes the following form:

$$t \leqslant 10^4 \text{ sec} \qquad (7)$$

When the water content is less, the inequality becomes still greater. Let us now assume that the water content of a cloud remains invariable. It is then possible to integrate (6) for an interval of time which will satisfy (7).

The expression $\frac{r^2}{Q_4(r)}$, where $Q_4(r)$ is a polynomial of the fourth degree in the curved brackets (6), can be factored into simple components by ordinary method. The roots of the polynomial, determined by Ferrari's method [5], prove equal to:

$$r_1 = 0{,}837 r_m, \quad r_2 = -0{,}809 r_m, \quad r_{3,4} = (-0{,}819 \pm i\sqrt{0{,}058})\, r_m.$$

After integration, we shall obtain for the time of growth of a droplet from r_0 to r (satisfying condition (7)):

$$t = \frac{363\eta}{g q_w r_m}\Big[0{,}16 \ln(r - 0{,}837 r_m) - 7{,}00 \ln(r + 0{,}809 r_m) + 3{,}42 \ln(r^2 + 1{,}639 r_m r + 0{,}729 r_m^2) + 3{,}44 \arctan\frac{2r + 1{,}639 r_m}{0{,}48 r_m}\Big]_{r_0}^{r}. \qquad (8)$$

The growth of the droplet for any interval of time in which Stokes' law of velocity is applicable can be obtained after calculating the change in r_m step by step in accordance with formula (2). In fact, unlike Findeisen, we consider effect of the growth of the fine droplets in the development of cloud or fog on the rate of growth of the enlarged droplet.

The results of calculations for the time of growth for water content $q_w = 1$ g/m^3 and for initial values of $r_{m0} = 5\mu$, $r_0 = 7\mu$ are shown in Table 1.

Table 1. Time Required for the Radius of a Droplet to Increase From r_0 to r

Stages of Radius (microns) Growth	7-8	8-9	9-10	10-11	11-13	3-15	15-20	20-25	25-30	30-40	40-50	50-70	70-100
Time of Growth (seconds)	720	570	470	380	585	330	540	390	190	285	240	250	190

Thus, the time required for an increase in size of an enlarged droplet from $r_0 = 7\mu$ to $r = 100\mu$ is approximately 1 hour and 25 minutes. During this same time, r_{m0} increases from 5μ only to 33μ. Figure 1 graphs the data of Table 1. The dotted curve shows the increase in r_m.

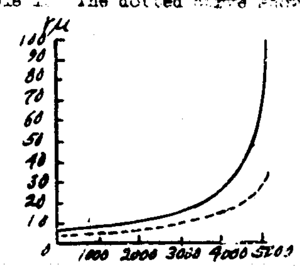

Figure 1. Growth of a droplet due to differences in Stokes' fall velocities. The unbroken line is the growth curve of an enlarged droplet; the dotted line is the growth curve of r_m.

Of course, for a cloud droplet to attain the size of a raindrop, it must fall for a long time within the cloud. This is made possible both by the great vertical size of the cloud and by the presence of ascending currents. Especially favorable conditions may arise in cumulus clouds during a prolonged flow of moisture to their base.

Contrary to the almost universal idea that the largest drops are always to be found at the base of a cloud, the opposite phenomenon takes place in rising cumulus clouds. The base of a cloud is composed of its very youngest part and retains only fine droplets which have not been able to grow larger. In the upper part of the cloud containing cloud masses with a longer life the average size of the droplets is larger and there must be individual droplets with radius differing greatly from that of the average mass of droplets.

Such a picture was actually observed in a microphotograph of droplets taken from an airplane in cumulus-type clouds in the summer of 1946 at the Main Geographic Observatory by I. I. Chestnaya and V. I. Zaytsev. In the crests of rising cumulus clouds of a size (vertically) of the order of 0.5 kilometers, they discovered individual droplets with radius 100 μ or more while the radius of droplets at the base of these clouds did not exceed 10 μ. The lifetime of the clouds studied amounted to 2-3 hours. The results of the microphotography are given in [6].

Consequently, our results agree satisfactorily with these experimental data, if the fact that we used a high value for the water content of a cloud be taken into acount.

It is also of interest to trace the temporal variation in the distribution curve of droplets with respect to size.

Figure 2 gives the distribution curves in conventional units:

$$n'(r)=\left(\frac{5}{r_m}\right)^4\left(\frac{r}{r_m}\right)^2 e^{\frac{5}{3}\left[1-\left(\frac{r}{r_m}\right)^3\right]} \qquad (1')$$

(radius is in microns) for certain values of r_m (5, 6, 8 μ), corresponding to various ages of the cloud mass.

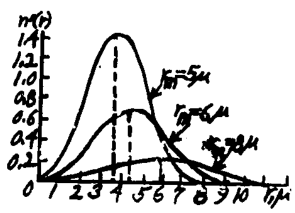

Figure 2. Distribution curves for [illegible] upon the age of the cloud.

The maximum of curve $n'(r)$ is determ[illegible] to zero the derivative with respect to r. We obtain the value

$$r'_m=\sqrt[3]{\frac{2}{5}}\,r_m\approx 0{,}74\,r_m \qquad (9)$$

For recently formed cloud masses (small values of r_m) the peak is sharp and the field of possible values for the radii is comparatively narrow. With increase in r_m, that is with the growth of the process of coagulation, the peak becomes flatter and the field of possible values for the radii increases.

The section of the curve characterizing the total number of droplets N diminishes rapidly.

N is easily calculated after integrating (1) with respect to all radii from 0 to ∞ for a given water content. We shall obtain:

$$N = \int_0^\infty n(r)\,dr = \frac{5 q_w}{4\pi r_m^3} = \frac{5}{3} N_m \qquad (10)$$

Where N_m is the number of droplets per 1 cubic centimeter in a homogeneous cloud of droplets of radius r_m.

Table 2 shows the number of droplets when there are two values for the water content, 1.00 and 0.16 grams per cubic meter(the latter value represents the average value of water content of clouds obtained experimentally by Diem [7].

Table 2. Total Number of Droplets per Cubic Centimeter of Cloud

Water Content in G per cu m	Number of droplets in 1 cm3 when r_m equals:									
	5	6	7	8	9	10	20	30	40	50 μ
1.00	3300	1600	1100	800	550	400	50	15	7	3
0.16	530	255	175	130	90	65	8	2.4	1.1	0.5

According to Houghton's data [8] for clouds, 1 cubic centimeter contains 50 to 500 droplets when the average value of r_m does not exceed 10μ. For fogs of a water content 0.05$\div$0.25 grams per cubic meter and with an average value of r_m = 20$\div$25μ, the number of droplets per cubic centimeter, according to the data of Houghton and Radford [3], equals 2$\div$6. Our data are in fair agreement with these results. The deformation of the form of the distribution curve with increase in the life of a cloud mass (taking into account the observations of droplet distribution in various parts of the cloud) also agrees with the concrete graphs of droplet distribution for various heights, as set forth in Diem's article [7] (see, for example, Flight Data, No 37, pp 61, 66).

The distribution curves of droplets, by size, plotted according to (1') have one maximum, the sharpness of which, as mentioned above diminishes with increase in r_m. But if the development of a given part of a cloud proceeds irregularly and an older cloud mass is intermingled with a younger mass, the droplets of which have not been able to grow larger, we can obtain a curve with two or more maxima, corresponding to the superposition of curves for various r_m's (see, for example, the distribution curve for droplets, according to size, for various N's in Diem's article [7]).

When the variation in water content is continuous (for instance, in the case of radiation fogs, with gradual drop in temperature) it is possible to calculate, for example, a linear time function according to the method given in this article by assuming q_w = constant for each stage and assigning a growth of q_w from stage to stage. We should then obtain for the distribution of droplets a more diffused curve than that according to formula (1), with increased asymmetry of the side of the large drops.

In conclusion, I express my deep gratitude to Ya. I. Frenkel for his valuable suggestions, and also to Ye. S. Seleznevaya and I. I. Chestnaya for their information on the results of microphotographing droplets.

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